Surds & Indices

(करणी एवं घात)

01. Laws of Indices (घातांक के नियम)

$(i).\ a^o = 1$

$(ii).\ (a^m)^n = a^{m\times n}$

$(iii).\ a^{m^n} \neq (a^m)^n$

$(iv).\ a^m \times a^n = a^{m+n}$

$(v).\ a^m \div a^n = a^{m-n}$

$(vi).\ a^m \div a^n \times a^p \times a^q \div a^k = a^{m-n+p+q-k}$

$(vii).\ a^{-m} = \frac{1}{a^m}$

$(viii).\ \left(\frac{a}{b}\right)^{-1} = \frac{1}{\frac{a}{b}} = \frac{b}{a}$

$(ix).\ a^m \times b^m = (ab)^m$

$(x).\ \frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m$

02. Laws of Surds (करणियों के नियम)

$(i).\ \sqrt[n]{a} = a^{\frac{1}{n}}$

$(ii).\ \sqrt[m]{n\sqrt{p\sqrt{a}}} = m \times n \times p\sqrt{a}$

$(iii).\ m\sqrt{a} \times m\sqrt{b} \times m\sqrt{c} = m\sqrt{a \times b \times c}$

$(iv).\ \frac{m\sqrt{a}}{m\sqrt{b}} = m\sqrt{\frac{a}{b}}$

$(v).\ \left(m\sqrt{n\sqrt{a}}\right)^o = 1$

$(vi).\ a\sqrt{2} + b\sqrt{3} = 5\sqrt{2} + 6\sqrt{3}$

$(vii).\ m\sqrt{a} = n\sqrt{a}$

$(viii).\ m\sqrt{(a^p)^q} = a^{\frac{pq}{m}}$

03. Some special rules (कुछ विशेष नियम)

Concept: 1

$$\sqrt{a\sqrt{a\sqrt{a\sqrt{a}}}\ldots\ldots\ldots \text{ n times}} = a^{\frac{2n^{-1}}{2n}}$$

Concept: 2

$$\sqrt{a\sqrt{a\sqrt{a\sqrt{a}}}\ldots\ldots\ldots} \propto = a$$

Concept: 3

$$\sqrt[n]{a\sqrt[n]{a\sqrt[n]{a\sqrt[n]{a}}}\ldots\ldots\ldots\ldots\infty} = a^{\left(\frac{1}{n-1}\right)}$$

Concept: 4

$$\sqrt[m]{a\sqrt[n]{b\sqrt[m]{a\sqrt[n]{b}}}\ldots\ldots\ldots\ldots\infty} = \sqrt[(mn-1)]{a^n b}$$

Concept: 5

$$\sqrt{a+\sqrt{a+\sqrt{a+\sqrt{a}}}\ldots\ldots\ldots\ldots\infty} = \frac{\sqrt{4a+1}+1}{2}$$

Concept: 6

$$\sqrt{a-\sqrt{a-\sqrt{a-\sqrt{a-\cdots\ldots\ldots\ldots\infty}}}}=\frac{\sqrt{4a+1}-1}{2}$$

Concept: 7

$$x=\sqrt{a+\sqrt{a+\sqrt{a+\sqrt{a+\cdots\ldots\ldots\ldots\infty}}}}$$

$$y=\sqrt{a-\sqrt{a-\sqrt{a-\sqrt{a-\cdots\cdots\cdots\cdots\infty}}}}$$

(i) $x+y=\sqrt{4a+1}$
(ii) $x-y=1$
(iii) $xy=a$

Concept: 8 & 9

$$p=\sqrt{a+\sqrt{a-\sqrt{a+\sqrt{a-\cdots\ldots\ldots\ldots\infty}}}}$$

$$q=\sqrt{a-\sqrt{a+\sqrt{a-\sqrt{a+\cdots\ldots\ldots\ldots\infty}}}}$$

(i) $p=\frac{\sqrt{4a-3}+1}{2}$
(ii) $q=\frac{\sqrt{4a-3}-1}{2}$

(iii) $p+q=\sqrt{4a-3}$

(iv) $p-q=1$

(v) $pq=a-1$

Concept: 10

$$\sqrt{a\div\sqrt{a\div\sqrt{a\div\sqrt{a\div\cdots\ldots\ldots\ldots\infty}}}}=a^{1/3}$$

Concept: 11

$$\sqrt{a+b\times\sqrt{a+b\times\sqrt{a+b\times\sqrt{a+\cdots\cdots\cdots\cdots\infty}}}}$$

$$=\frac{\sqrt{4a+b^2}+b}{2}$$

Concept: 12

$$\sqrt{a-b\times\sqrt{a-b\times\sqrt{a-b\times\sqrt{a-\cdots\ldots\ldots\ldots\infty}}}}$$

$$=\frac{\sqrt{4a+b^2}-b}{2}$$

Concept: 13

$$m=\sqrt{a+b\times\sqrt{a+b\times\sqrt{a-\cdots\ldots\ldots\ldots\infty}}}$$

$$n = \sqrt{a - b \times \sqrt{a - b \times \sqrt{a - \cdots\cdots\cdots\cdots \infty}}}$$

(i) $m + n = \sqrt{4a + b^2}$
(ii) $m - n = b$
(v) $mn = a$

Concept: 14

1+ Consecutive Series

$$\sqrt{1 + 2\sqrt{1 + 3\sqrt{1 + 4\sqrt{1 + 5\sqrt{1 + 6 \ldots \ldots \infty}}}}}$$

3 - Ans

Concept: 15 (Power Concept)

$$\sqrt{a}^{\sqrt{a}^{\sqrt{a}^{\sqrt{a}\ldots\ldots\infty}}} = \frac{1}{8}$$

$$(\sqrt{a})^{\frac{1}{8}} = \frac{1}{8}$$

$$\left(a^{\frac{1}{16}}\right)^{\frac{1}{16}} = \left(\frac{1}{8}\right)^{\frac{1}{16}} \Rightarrow a = \left(\frac{1}{8}\right)^{16}$$

$$\Rightarrow a = \frac{1}{8^{16}}$$

Concept: 16 (Fix Value Series)